UDALA (JR.) COLLEGE, UDALA
TEST EXAM -2018
SUB :MATHEMATICS
CLASS : +2 2nd YEAR SCIENCE/ ARTS

TIME: 2 HOURS F. M. – 100

[Right-hand margin indicates marks]

GROUP -A

1. Answer all these questions: [10 x 1]

i) If $y = [x]$, find $\frac{dy}{dx}$, at $x = 0$

ii) Find the point where $e^{|x|}$ is not differentiable .

iii) Give example of a function $y = f(x)$ such that

$$\frac{d^3y}{dx^3} = y$$

iv) If $Z = \sin^{-1}\left(\frac{x}{y}\right)$, what is the value of $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$?

v) What is the value of $\lim_{x\to\infty} \frac{x^n}{e^x}$; $n \in N$.

vi) What is the value of $\int f(x)\, f'(x)dx$.

vii) Write the value of $\int \frac{e^x(1+x)dx}{\cos^2(xe^x)}$.

viii) Write the equation of the plane which passes through the point (1, 1, 1,) and its direction cosines are <-1, 1, -1> .

ix) Write the centre of the sphere which touches the coordinate planes XY, YZ and ZX.

x) If $\vec{a}, \vec{b}$ and $\vec{c}$ are linearly independent vectors, then what is the value of $[\vec{a}, \vec{b}, \vec{c}]$.

P.T.O.

GROUP -B

2. Answer **any four** of the following; [5 x 4]

a) Prove that $\frac{d}{dx}\left[\frac{1-\tan x}{1+\tan x}\right]^{1/2} = \frac{-1}{\sqrt{\cos 2x}\,(\cos x+\sin x)}$.

b) Differentiate $\tan^{-1}\sqrt{\frac{1-t}{1+t}}$.

c) If $y = e^{x^{x^{\cdots\cdots^{\infty}}}}$, then find $\frac{dy}{dx}$.

d) Write the tangent equation to the parabola $y^2 = 4ax$ at the point$(at^2, 2at)$.

e) Write the open interval in which the function $f(x) = x^{1/x}$; $for\ x > 0$ is decreasing.

f) Evaluate the $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$.

3. Answer **any four** of the following; [5 x 4]

a) Evaluate $\int \frac{x^4}{1+x^2}dx$.

b) Evaluate $\int \sqrt{\frac{a-x}{a+x}}dx$.

c) Evaluate $\int sin^4 x\,dx$.

d) Evaluate $\int \frac{xe^x}{1+x^2}dx$.

e) Solve $\frac{dy}{dx} + y\tan x = \sec x$.

f) Find the area common to curves $y = \sqrt{x}$ and $x = \sqrt{y}$

4. Answer **any four** of the following; [5 x 4]

a) If a variable plane meets the coordinate axes at A,B,C and is at a constant distance 'd' from the origin, then prove that the lows of the centaid of the triangle ABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{9}{d^2}$

b) Write the symmetric form of equation of the line $x + 2y + z - 3 = 0 = 6x + 8y + 3z - 10$.

c) Prove that $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$

d) Prove by vector method that an angle inscribed in a semi-circle is right angle.

e) Show the feasible region for the LPP.

$$x + y \geq 1\ ,\ 2x + y \leq 4 \text{and} \geq 0, y \geq 0\ .$$

f) Prove that 4- points whose position vectors are $4\hat{\imath} + 5\hat{\jmath} + \hat{k}$, $-\hat{\jmath} - \hat{k}$, $3\hat{\imath} + 9\hat{\jmath} + 4\hat{k}$ and $-4\hat{\imath} + 4\hat{\jmath} + 4\hat{k}$ are coplanar.

GROUP -C

5. Answer **any four** of the following; [7.5 x 4]

a) Use the function $f(x) = x^{1/x}\ ; x > 0$ to prove that $\pi^e < e^\pi$.

b) If $y = \frac{x}{x^2+a^2}$ then prove that $y_n = \frac{(-1)^n n!\ \cos(n+1)\theta}{(x^2+a^2)^{\frac{n+1}{2}}}$.

c) If a variable plane is at a constant distance P from the origin, which meets the coordinate planes XY, YZ and ZX at A, B, C respectively. Then show that the laws of their points of intersection is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$.

d) Find the distance of the point $(1, -1, -10)$ from the line $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ measured parallel to the line $\frac{x+2}{2} = \frac{y-3}{-3} = \frac{z-4}{8}$.

e) Solve, $(1 + y^2)dx + xdy = tan^{-1}ydy$.

f) Evaluate $\int \frac{dx}{1+x^4}$

g) Prove that $\int_0^{\pi} x \ln \sin x \, dx = \frac{\pi^2}{2} \ln \frac{1}{2}$

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